

# Unconditionally secure one-way quantum key distribution using decoy pulses

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## Abstract

We report here a complete experimental realization of one-way decoy-pulse quantum key distribution, demonstrating an unconditionally secure key rate of 5.51 kbps for a 25.3 km fibre length. This is two orders of magnitudes higher than the value that can be obtained with a non-decoy system. We introduce also a simple test for detecting the photon number splitting attack and highlight that it is essential for the security of the technique to fully characterize the source and detectors used.

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Communication with single photons promises for the first time a method to exchange digital keys between two remote parties (referred to as Alice and Bob),<sup>1,2,3</sup> the secrecy of which are not reliant upon assumptions about an eavesdropper's surveillance equipment, computing power or guile.<sup>4,5</sup> Currently, however, the security of quantum key distribution (QKD) is compromised by the multi-photon pulses inevitably produced by the attenuated lasers used in today's systems.<sup>6,7</sup> In the photon number splitting (PNS) attack, an eavesdropper (Eve) preferentially transmits these multi-photon pulses to Bob through a lossless channel after removing part for measurement, while blocking a corresponding number of single photon pulses to maintain the same detection rate at Bob.<sup>8</sup> By doing so, she can determine part, or all, of the key while remaining hidden.

Recently it has been proposed that the PNS attack may be foiled using decoy pulses, with a different average intensity from the signal pulses.<sup>9,10,11,12</sup> By determining the transmission of the signal and decoy pulses separately, Alice and Bob are able to detect PNS attacks. An early experiment<sup>13</sup> has already demonstrated the feasibility of the decoy pulse protocol, although using a send-and-return architecture for which Alice does not have full control of the individual pulse intensities, as required for its secure implementation. We report here a complete experimental demonstration of one-way decoy pulse QKD over 25.3 km fibre with a continuous key rate of 5.51 kbps, which is unconditionally secure against all types of attacks, including PNS.

Decoy pulse QKD theory gives a rigorous bound of the characteristics of the single photon pulses, which are the only source pulses that contribute to the secure bit rate.<sup>14,15</sup> In the simplest decoy protocol,<sup>12</sup> two different average pulse intensities (referred to as signal and decoy pulses) with mean photon numbers of  $\mu$  and  $\nu$  ( $\mu > \nu$ ) respectively are used. By measuring the transmittances  $Q_\mu$  and  $Q_\nu$ , *i.e.*, Bob's detection probability of a signal or decoy pulse respectively, the lower bound ( $Q_1^L$ ) of the single photon transmittance, which is defined as the joint probability that a signal pulse contains only one photon and the pulse is detected by Bob, can be written as<sup>12,13</sup>

$$Q_1^L = \frac{\mu^2 e^{-\mu}}{\mu\nu - \nu^2} (Q_\nu^L e^\nu - Q_\nu e^\mu \frac{\nu^2}{\mu^2} - \varepsilon_\mu Q_\mu e^\mu \frac{\mu^2 - \nu^2}{\frac{1}{2}\mu^2}) \quad (1)$$

where  $\varepsilon_\mu$  is the bit error rate of the signal pulses, and  $Q_\nu^L$  is the lower bound of the decoy pulse transmittance, estimated conservatively as 10 standard deviations of  $Q_\nu$  from the measurement value, ensuring a confidence interval of  $1 - 1.5 \times 10^{-23}$ . By assuming all bit

errors are from single photon pulses, the upper bound,  $\varepsilon_1^U$ , for the bit error rate of the single photon pulses can be bounded as

$$\varepsilon_1^U = \frac{\varepsilon_\mu Q_\mu}{Q_1^L} \quad (2)$$

With the bounds  $Q_1^L$  and  $\varepsilon_1^U$  available, the secure bit rate is readily calculated, which will be shown later.

We use a one-way fibre-optic QKD system with phase encoding, schematically shown in Fig. 1. It uses two asymmetric Mach-Zender interferometers for encoding and decoding. The sending (Alice) and receiving (Bob) units are connected with a 25.3 km fibre spool. Both the signal and decoy pulses are generated by a 1.55  $\mu\text{m}$  pulsed laser diode of fixed intensity operating at 7.143 MHz. The pulses are modulated with an intensity modulator to produce the desired ratio of signal pulse strength to decoy pulse strength, and are then strongly attenuated to the desired level by an attenuator before leaving Alice's apparatus. Synchronization is realized by multiplexing a 1.3  $\mu\text{m}$  clock signal with the quantum signals so that clock and quantum signals can be sent through the same fibre. An active stabilization technique<sup>16</sup> is used to ensure continuous operation.

Two single photon detectors based on InGaAs avalanche photodiodes are carefully prepared for the experiment. Care was taken to minimize detector afterpulsing,<sup>17</sup> which correlates photon detection events of the signal and decoy pulses and prevents faithful measurements of  $Q_\mu$  and  $Q_\nu$ . The system detection efficiency, including channel transmission loss (4.7 dB) and Bob's loss (2.5 dB), was characterized at  $1.95 \times 10^{-2}$ . The sum of the dark count rates for the two detectors was  $9.4 \times 10^{-5}$  per pulse. Custom electronics were developed for driving the QKD optics.

We implemented the one weak decoy state + BB84 protocol<sup>12</sup> in our experiment. The decoy pulse probability is chosen to be 1/4. Each QKD session is set to be  $1 \times 10^6$  sifted bits, including both signal and decoy pulses, corresponding to approximately  $2 \times 10^6$  photon detection events. The signal and decoy pulse intensities are chosen as  $\mu = 0.425$  and  $\nu = 0.204$  to provide optimum secure bit rate.

We recorded the output from the decoy QKD experiment continuously for four hours, during which a total of 372 QKD sessions were executed, each lasting 38.7 s on average. The measured bit error rates ( $\varepsilon_\mu, \varepsilon_\nu$ ) for signal and decoy states, as shown in Fig. 2(a), are stable at 1.72% and 2.74% respectively. These values are consistent with the measured

interferometer visibility, mis-compensation, and detector dark counts. Transmittances  $Q_\mu$  and  $Q_\nu$ , shown in Fig. 2(b), fluctuate in tandem, and such fluctuation is attributed mainly to variation of the channel transmission and the detection efficiency. The sharp dips in  $Q_\mu$  and  $Q_\nu$  are due to polarization or phase resets of the active alignment system, during which photons were temporarily not counted. Figure 2(c) shows the measured ratio of  $Q_\mu/Q_\nu$ , which remains virtually constant despite the larger fluctuations in  $Q_\mu$  and  $Q_\nu$ . The measured ratio is remarkably consistent with the theoretically expected (red line), indicating the system has produced and detected both signal and decoy pulses reliably.

The final secure bit rate,  $R$ , was obtained, after estimating  $Q_1^L$  and  $\varepsilon_1^U$  using Eqns. (1) and (2), by first applying a modified Cascade<sup>18</sup> error correction routine, followed by privacy amplification,<sup>19</sup> using

$$R \geq \{-N_\mu^{Sift} f_{ec} H_2(\varepsilon_\mu) + N_1^{Sift} [1 - H_2(\varepsilon_1^U)]\}/t \quad (3)$$

where  $N_\mu^{Sift}$  is the number of the sifted bits from the signal pulses,  $N_1^{Sift} = \frac{1}{2}Q_1^L t$  the lower bound of the number of the sifted bits for the single photon pulses, and  $t$  is the QKD session time.  $H_2(\varepsilon) = -\varepsilon \log_2(\varepsilon) - (1 - \varepsilon) \log_2(1 - \varepsilon)$  is the binary Shannon entropy.  $f_{ec}$  is the error correction efficiency relative to Shannon limit. Our error correction module gives  $f_{ec} \approx 1.10$  for the given bit error rate.

As shown in Fig. 3, the secure bit rate remains fairly stable and positive for all decoy sessions, with an average value of 5.51 kbps. The rate is much higher than a non-decoy QKD system. For a non-decoy system using the same detectors, we found the maximum fibre length secure from the PNS attack to be just 9 km. With quieter detectors, a secure bit rate of 43 bps was previously achieved for 25 km of fiber.<sup>20</sup> Thus, our implementation of the decoy protocol has allowed more than two orders of magnitude enhancement in the secure bit rate.

To detect the PNS and related attacks, Alice and Bob can monitor the ratio  $Q_\nu/Q_\mu$ . In the current experiment, we expected a ratio of  $Q_\nu/Q_\mu \doteq \frac{\nu\eta+Y_0}{\mu\eta+Y_0} = 0.486$ , where  $\eta$  is the system detection efficiency and  $Y_0$  is the dark count rate. Significant deviation of the measured ratio from this expected value indicates a PNS by Eve. In the PNS attack, Eve first counts the number of photons in each pulse using a quantum non-demolition measurement and preferentially transmits the multiphoton pulses to Bob. Since the signal pulses are stronger than the decoy pulses, and thus contain more multi-photon pulses, the PNS will

tend to reduce  $Q_\nu/Q_\mu$ . Figure 4 shows a simulation, together with the experimental points, of the secure bit rate dependence on the ratio  $Q_\nu/Q_\mu$ . The experimental data is in excellent agreement with theory when the PNS is not present. With the PNS attack present, the simulated secure bit rate decreases rapidly with the ratio  $Q_\nu/Q_\mu$ . No secure bit rate is possible when the ratio is merely 17% smaller than expected, highlighting the privacy amplification cost to remove information leaked by the PNS attack. Such a sensitive dependence puts stringent requirements on the preparation and measurement of the signal and decoy pulses. The measured ratio may be slightly higher than expected due to statistical uncertainty, or if Eve chooses to preferentially transmit the single photon pulses. A more likely case of the measured ratio exceeding the expected value is due to artifacts, such as mis-calibration of signal and decoy intensity ratio and/or detector afterpulsing. Such artifacts would cause an over-estimation of secure bit rate, thereby compromising security, and must therefore be carefully eliminated. This has been achieved in our experiment by careful calibration of the source and detectors.

Secure bit rate is dependent on the session duration due to the statistical certainty with which  $\varepsilon_1^U$  and  $Q_1^L$  can be estimated. Longer sessions allow a more precise determinate and a higher bit rate. It can be seen in Fig. 5 that the calculated secure bit rate increases with session duration, approaching the maximum value for sessions exceeding 1000 seconds. Due to statistical fluctuations, there is a minimum session time for establishing a secure bit rate, found here to be around 0.4 s. We perform decoy protocol QKD at three distinct session times: 9.2, 18.5, and 38.7 s, and results are shown also in Fig. 5. Even with the shortest time of 9.2 s, a secure bit rate of 4.80 kbps is achieved, which is about 78% of the saturation rate with infinite session time, demonstrating that long session times are not necessary for decoy pulse QKD.

In summary, a practical one-way decoy QKD system has been demonstrated over a 25.3 km fiber with an average bit rate of 5.51 kbps. It is unconditionally secure against all types of attacks, including the PNS attack. We conclude that decoy pulses improve the security and performance of weak pulse QKD. However, sources and detectors must be calibrated accurately to avoid any artifacts that may compromise security.

During preparation of this manuscript two other implementations of decoy pulse QKD have appeared.<sup>21,22</sup> These show operation over longer fibre spools but with much lower key generation rates and are limited to local synchronization.

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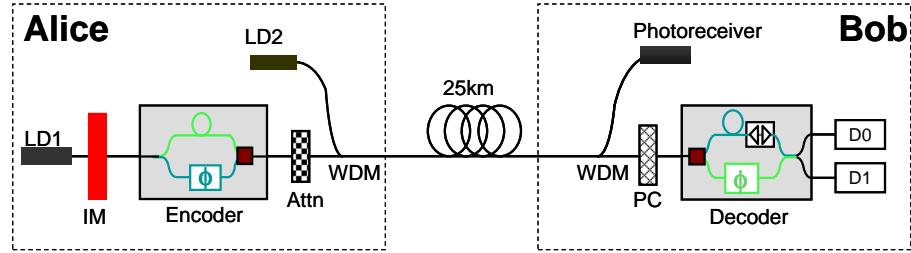


FIG. 1: Schematics of optical layout of the one-way decoy quantum key distribution system using BB84 phase-encoding; Attn: Attenuator; IM: intensity modulator; LD1: signal laser diode at  $1.55\mu\text{m}$ ; LD2: clock laser diode at  $1.3\mu\text{m}$ ; PC: polarisation controller; WDM: wavelength division multiplexer; D0, D1: single photon detectors.

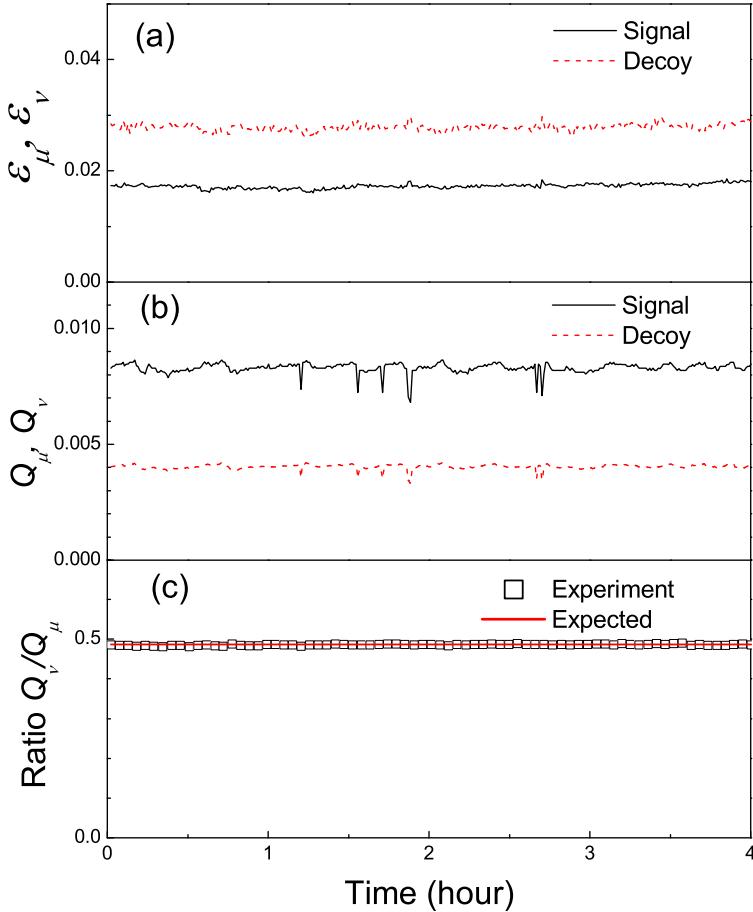


FIG. 2: Experimental results of a continuous series of decoy-QKD sessions for four hours. (a) Bit error rates measured for the signal and decoy pulses; (b) The measured transmittances for the signal pulses ( $Q_\mu$ ) and decoy pulses ( $Q_\nu$ ); (c) The measured ratio  $Q_\mu/Q_\nu$  (empty squares) in comparison with the expected value (line).

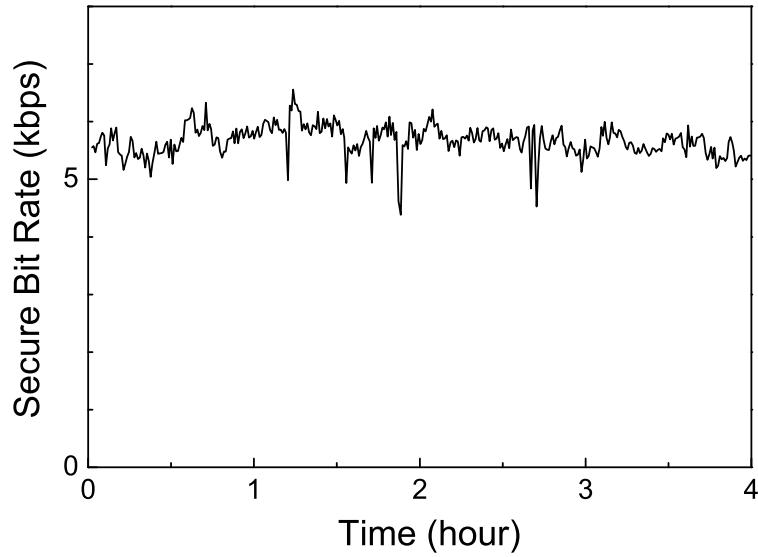


FIG. 3: Secure bit rate as a function of time.

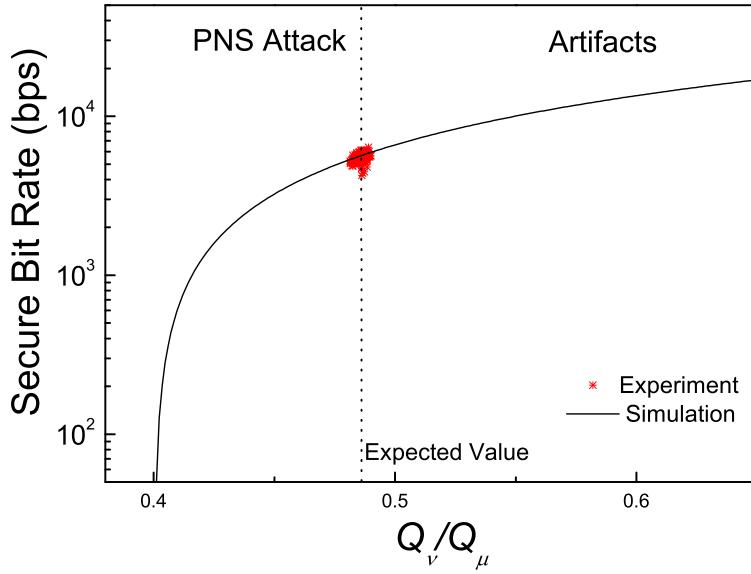


FIG. 4: Simulated (solid line) and experimental (symbols) secure bit rate dependence on the transmittance ratio  $Q_\nu/Q_\mu$ . The vertical dotted line indicates the expected ratio.

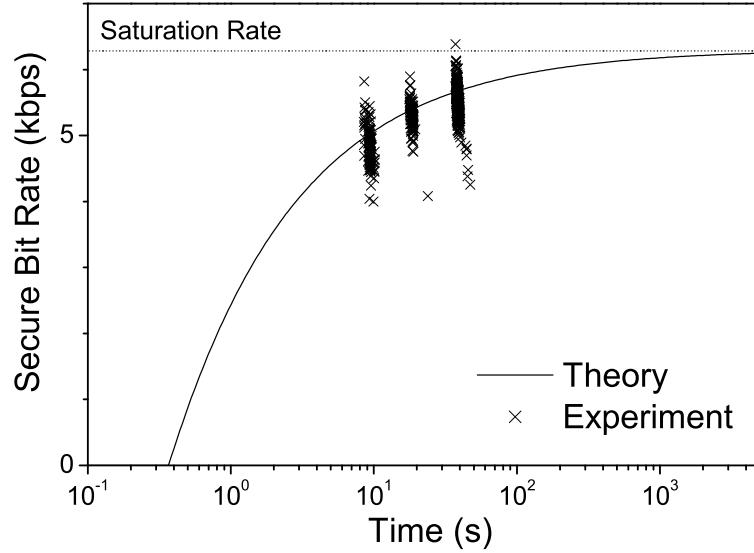


FIG. 5: Simulated (solid line) and experimental (symbols) secure bit rate as a function of QKD session duration. The dotted line shows the saturation secure bit rate with infinite QKD session time.